Regulation Under Stock Market Information Disclosure *

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Abstract

It is known that stock prices of public listed regulated companies react to price revisions by the regulator, and that the information conveyed by this price reaction might be used by the regulator on the contract design. This paper builds on Laffont and Tirole (1986)’s regulation model with observable costs to better understand the effects the inclusion of the stock market can have on the regulator-regulated firm relationship. Our numerical simulations show that the inclusion of the market induce more powerful incentive schemes, with higher cost-reducing efforts, smaller informational rent by the firms and higher overall social welfare. In particular, we find that when the regulator is committed, the presence of the market can make the first-best contract feasible, and that in the non-commitment case the market affects the firm’s strategy by making it reveal more information about its cost than it normally would.

1 Introduction

The relationship between a regulator and a regulated firm has been the subject of various studies. The new economics of regulation, which has applied the principal-agent methodology to modelling the contractual relationship between regulators and regulated firms, has provided many important insights into how this interaction takes place. However, many other factors, external to the regulation per se, such as political aspects or the firm’s governance structure, can impose significant constraints and influence this interaction. This article is concerned with one specific factor which we believe can alter the interaction between regulator and regulated firm: the public listing of the regulated firm.

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1See Laffont (1994).
In practice, several examples can be found where the regulator has somehow reconsidered or revised an existent regulatory policy after observing the performance of regulated companies in the stock market. One of the most striking examples of such revision was the case of the UK electricity regulator in the first periodic tariff revision in 1994, as mentioned in Faure-Grimaud (2002). Also in the electricity sector, the recent methodology changes implemented by the Brazilian regulator, Aneel, for the third tariff review cycle (RTP\(^2\)) have generated widespread complaints from the regulated companies, and raised some questions about the stability of the rules and the regulator’s commitment. We conjecture that these changes can be at least partially attributed to the distribution companies’ positive market performance.

Previously, during the second RTP, even though the market’s opinion on the tariff adjustment announced usually ranged from neutral to slightly negative, the performance of most of the companies in the quarter following the adjustment was positive, which could indicate that those adjustments were not too strict. On the other hand, when the changes in the procedures for the third RTP were announced, reports showed the market’s concern on the companies’ ability to maintain their profitability, and some analysts recommended a cautious approach to the sector for the subsequent years. While it is clear that the regulator’s decisions affect the market stock price, because of its direct effect on the companies’ current profit, the adjustments in the methodology for the third RTP indicates that the regulator’s decisions may also be affected by the information revealed by the market prices.

In this context, the question that arises is whether, and how, the public listing of a company in the market can have an impact on the regulator-regulated firm relationship. In this paper, we examine some aspects of this relationship that may be affected. The basic idea underlying our work is that firms may want to send different signals to the regulator and to its investors. Publicly traded companies are required to provide information to the market on their operational performance and financial statements on a regular basis, and this information is observable to other parties – particularly, it is observable to the regulator. Since market prices usually react to this information, traded firms have a desire to demonstrate that they are efficiently managed, which usually means showing low costs and high profitability to their shareholders.

On the other hand, it is well established in the theory of regulation that the relationship between the regulator and the regulated firm is built based on the assumption that the

\(^2\)For more details on the changes for the third tariff review cycle see “Resolução Normativa nº 457/2011, de 08/11/2011” and “PRORET - Procedimentos de Regulação Tarifária”, on Aneel’s website (www.aneel.gov.br).

regulator is at an informational disadvantage as to the firm’s operational structure and cost figures. As such, the firm is reluctant to reveal too much information about its cost structure, fearing that this information will be used by the regulator in the future to impose harsher efficiency goals. In a static environment, or when the regulator can commit to a long-term contract, the firm reveals itself in exchange for an informational rent. However, if the regulator is not committed, there is no guarantee that she will not expropriate the firm once its efficiency is revealed.

In practice, it is quite easy to find examples where stock prices of listed regulated companies reacted to regulators’ announcements of price revisions. While not as common, the regulator may also react to stock prices: the notion that market prices convey information about the firm’s operational performance seems undisputed. Furthermore, it is clear that the firms’ decision are affected by the market presence. The basic purpose of our model is thus to understand whether the presence of the market can affect the firm’s behaviour towards the regulator and the contracts offered by the regulators to the firm.

1.1 Related Literature

Some previous work has given attention to the impact of public listing in regulation. Faure-Grimaud (2002) examines the role of the stock prices in the regulation of firms and is perhaps the most closely related our work. The article builds on a regulation model where the regulator defines a price cap depending on her knowledge about the firm’s cost parameter – which depends on the regulator’s costly monitoring technology. The firm is publicly traded, and stock prices can be informative of the firm’s value. Particularly, the information collection by the regulator and the stock price information are substitutes. Thus, there will be less monitoring by the regulator when the firm is publicly listed.

In Faure-Grimaud (2002), the regulator is unable to commit to long-term regulation. Due to this lack of commitment, the presence of the market can make it easier for the regulator to obtain information about the firm and expropriate its profit. The present article builds on the same motivation. However, differently from Faure-Grimaud (2002), we endogenize the disclosure of information by the firm through the introduction of a signalling game, where the manager of the firm actively chooses the signal he will divulge to the market – and, consequently, to the regulator. Further, in our case the regulator does not have access to any particular monitoring technology, thus the only information available to both the regulator and the market is the public signal sent by the firm.
To model the firm’s signalling game, we build on Miller and Rock (1985). In their study, the authors analyze the effects of dividend announcements on the market’s perception of the firm’s profitability under asymmetric information. With this objective, they build a signalling model where firms use the payment of dividends, paid at the expense of forgone investments, in order to signal their future profitability and consequently affect their short-term market value.

One important aspect of their model, which is also incorporated in ours, is the assumption that there are two types of investors in the firm: the “insiders”, represented by the companies’ managers and better informed about the firm’s real intrinsic value, and the “outsiders”, who do not observe the firm’s real value, trading their stock based on the firm’s perceived value after the dividend announcement. Thus, there is a fraction of the shares which are owned by outsiders, while the remainder of the shares belong to insiders – and the objective function of the firm incorporates their respective weights, proportionally to the value of their holdings. As a consequence, the manager uses the dividend announcement in order to manipulate the market expectation, deviating from the investment level that maximizes the present value of the firm’s future earnings.

Di Tella and Kanczuk (2003) also incorporate market information to regulation. They propose a simple linear mechanism to punish regulated firms who outperform the market during rate reviews periods, as a way to prevent cost padding. However, their focus is mainly on the benefits of using stock prices to reduce the information problem, and not on the commitment problem.

Other works acknowledge the ability of market prices to convey information on firms. Particularly, Bond, Goldstein, and Prescott (2010) do so in the context of analyzing the effect of government policies that aim to help firms in trouble on the informational content of market prices. Lehar, Seppi, and Strobl (2011) perform a similar analysis in the context of banking regulation, but focusing on the theoretical foundations of market-based bank regulation. Despite being based on the central idea of analyzing the interaction between market prices and (some type of) government intervention in firms, both articles are developed in a different context from ours, and are not directly related to our work.

As previously stated, our objective is to understand how the the presence of a third agent (the market) alters the relationship between the regulator and the regulated firm. For that, we build on Laffont and Tirole (1986)’s regulation model with observable costs. Specifically, we model the trade-off the firm faces when sending the cost signal by assuming

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3 For this reason, in Miller and Rock (1985) outside investors are viewed as short-term investors, while insiders are viewed as long-term investors.
that the firm’s manager maximizes a weighted function of the firm’s real profit and its short-term market value, as in Miller and Rock (1985). Our main contribution is that the level of information disclosure of the firm is endogenous in our model. We analyse both the case where the regulator is committed, and thus the firms always reveals its efficiency parameters without the market presence, and the no-commitment case (Laffont and Tirole, 1988), which leads to some degree of pooling.

Our results indicate that the presence of the market does indeed have a strong impact on the relationship between the regulator and the firm. It increases welfare in both cases, while inducing more separation of types, and leading to more powerful incentives. Particularly, we find that in the commitment case the optimal asymmetric information outcome approaches the first-best as the market presence increases.

This paper is organized as follows. In a preliminary analysis, we present a basic regulation model for both the full information and asymmetric information cases without the market presence, which is the basis for our model, and introduce a new agent to this set up: the short-term investors in the stock market. In Section 3, we develop our model and discuss the benchmark commitment and non-commitment cases. Section 4 presents our numerical results, while Section 5 concludes.

2 Preliminary analysis

In this section, we introduce our problem by presenting a simple, static regulation model. After introducing the main elements of the model, we solve the static, full information case and then the static problem under imperfect information. This model is presented in Laffont and Tirole (1986). We also introduce another important addition of our model, the market.

2.1 Full Information

Suppose a regulator who wishes to provide some type of public service with value \( S > 0 \) to consumers. Consider a model where a firm capable of providing such service has cost function:

\[
C = \theta - e,
\]
where $\theta$ is an exogenous parameter of efficiency of the firm and $e$ is an (endogenous) effort that the manager can exert to reduce costs. Exerting effort has a cost, given by $\psi(e)$.

The regulator makes net monetary transfers $t$ to the firm and costs are fully reimbursed. We assume that transfers to the firm can only be funded by taxes and, due to distortionary taxation, there is a cost $\lambda > 0$ incurred by the planner for collecting public funds, such that each unit of transfers to the firm costs $(1 + \lambda)$ to the regulator. Thus, consumer’s welfare is

$$S - (1 + \lambda)(t + C),$$

and the firm’s utility level is given by

$$U = t - \psi(e),$$

which implies that $t = U + \psi(e)$.

In the full information setting, the regulator observes both the cost and the parameter $\theta$. Her problem is then to choose a pair $(t, e)$ to maximize the welfare function

$$W^{FI} = S - (1 + \lambda)(t + \theta - e) + U$$

subject to

$$U \geq 0. \quad (2)$$

The individual rationality constraint (2) says that the utility level of the firm’s manager must be non-negative for the firm to participate in the contract. In other words, the utility derived by the manager from the contract should be higher than its opportunity cost (which we normalize to zero). The optimal solution for the problem is then

$$t = \psi(e)$$

$$e = e^*, \text{ where } \psi'(e^*) = 1.$$ 

This function satisfies the usual properties: $\psi'(e) > 0$ and $\psi''(e) > 0$ for $e > 0$, with $\psi(0) = 0$ and $\lim_{e \to 0} \psi'(e) = 0$.

Total transfers made by the regulator to the firm are $T = t + C$. 

\footnote{This function satisfies the usual properties: $\psi'(e) > 0$ and $\psi''(e) > 0$ for $e > 0$, with $\psi(0) = 0$ and $\lim_{e \to 0} \psi'(e) = 0$.}
2.2 Asymmetric Information

Under imperfect information, the regulator can only observe the firm’s cost \( C \) - but not its decomposition between the firm’s inherent cost \( \theta \) and effort \( e \). We assume the parameter \( \theta \in \{\theta_L, \theta_H\} \), with \( \theta_H > \theta_L \), is known only to the firm’s manager, and the other agents in the economy know only its prior distribution \( P_{\theta} [\theta = \theta_L] = v^1 \).

The contract offered to the firm by the regulator is a pair \((t, C)\), where the firm receives a transfer \( t(C) \) according to the cost observed. According to the Revelation Principle, the regulator can restrict herself to direct, truthful mechanisms \((t(\theta), C(\theta))\). For notational simplicity, let us make \( t_L \equiv t(\theta_L), C_L \equiv C(\theta_L) \), and so forth.

The rent of a firm of type \( \theta \) when it selects the contract designed for its type is

\[
U(\theta) = t(\theta) - \psi(\theta - C(\theta)),
\]

and, also for notational simplicity, \( U_L = t_L - \psi(\theta_L - C_L) \) and \( U_H = t_H - \psi(\theta_H - C_H) \).

It is the incentive compatibility constraint that guarantees that each firm chooses the contract designed for its own type in the menu of contracts. For the static problem under imperfect information, those constraints are:

\[
t_L - \psi(\theta_L - C_L) \geq t_H - \psi(\theta_L - C_H) \tag{3}
\]

\[
t_H - \psi(\theta_H - C_H) \geq t_L - \psi(\theta_H - C_L) \tag{4}
\]

Similarly, individual rationality for this case amounts to

\[
U_L \geq 0 \tag{5}
\]

\[
U_H \geq 0 \tag{6}
\]

The regulator will solve the objective function

\[
W^{AI} = E[\theta] [S - (1 + \lambda) [C(\theta) + \psi(\theta - C(\theta))] - \lambda U(\theta)],
\]

subject to the incentive and individual rationality constraints for both types, shown above in equations (3)-(6). The solution to this problem is\[^6\]

\[^6\] Notice that \( e = \theta - C \).
\[ \psi'(e_L) = 1 \quad (\text{or } e_L = e^*) \]  
\[ \psi'(e_H) = 1 - \frac{\lambda v}{(1 + \lambda)(1 - v)} \Phi'(e_H), \]  

where \( \Phi(e_H) \equiv \psi(e_H) - \psi(e_H - \Delta \theta) \), with \( \Delta \theta = \theta_H - \theta_L \), is the informational rent obtained by the efficient firm.

These results illustrate the basic trade-off faced by the regulator, namely, that in order to induce more effort from the inefficient firm, the regulator has to leave more informational rent to the efficient one.

### 2.3 Market Signalling

In addition to the two agents described above, the firm and the regulator, we now introduce a third agent: the short-term investors in the stock market (henceforth, the market). As in \[\text{Miller and Rock (1985)}\], we suppose that there exists an asymmetry of information between the firm’s long-term investors (such as managers and directors of the firm), who are considered insiders and have private information about its costs, and the short-term investors, who have no private information and are considered outsiders to the firm.

This asymmetry of information leads to differences in the perceived value of the firm for the different types of investors. The insiders are interested in maximizing the firm’s profits, defined as a function of the true inherent cost \( \theta \) and the chosen contract \( \tilde{\theta} \)

\[ U(\theta, \tilde{\theta}) = t(\tilde{\theta}) - \psi(\theta - C(\tilde{\theta})). \]

On the other hand, the outsiders are interested in maximizing the market value of the firm. A regular assumption about the behaviour of the stock market is that, if all the participants are equal and have the same information, no expected profit can be made from trading. Assuming that long-term investors plan on keeping their stocks, the market value of a firm is defined as a function of the chosen contract by

\[ U^m(\tilde{\theta}) = E_{\theta}[U(\theta, \tilde{\theta})|C = C(\tilde{\theta})], \]

\footnote{Note that we focus on direct mechanisms, and therefore contracts \((t(\hat{\theta}),C(\hat{\theta}))\) can be represented by \(\hat{\theta}\). When \(\theta \neq \hat{\theta}\), however, the firm’s profit depends on \(\hat{\theta}\) only through the contract associated to it.}
which states that the market value of the firm is given by what short-term investors believe is the firm’s profit, given the observed chosen contract. If the market believes each firm selects its own contract, as in the basic model depicted in the previous sections, the firm’s market value when choosing its contract would be its real profit. However, differences may arise if the market believes the firms randomize.

The presence of short-term investors generates a potential conflict of interests between stockholders, since the best decision for increasing profit may not be the one that maximizes the market value, and vice-versa. For example, even if the two contracts yield the same profit for the inefficient firm, choosing the other type’s contract can maximize market value since the market may believe that the firm is efficient with positive probability. Therefore, the firm’s objective function should take into account this conflict, so that neither type of investors have incentives to bribe the managers to depart from the firm’s optimal strategy.

Following Miller and Rock (1985), we use as objective function a simple “social welfare function”, in which the manager attaches weights to each group’s utility based on its proportion of holdings. Thus, if the fraction of long-term investors is $k$, the manager’s objective function can be written as

$$U(\theta, \tilde{\theta}) = kU(\theta, \tilde{\theta}) + (1-k)U^m(\tilde{\theta})$$

$$= kU(\theta, \tilde{\theta}) + (1-k)E\theta[U(\theta, \tilde{\theta})|C = C(\tilde{\theta})].$$

For notational ease, for the rest of this paper, $U(\theta, \theta)$ and $U(\theta, \tilde{\theta})$ will be denoted by $U(\theta)$ and $U(\tilde{\theta})$.

3 The Model

Our model builds on the basic, static model presented in Section II by introducing a second period, as in Laffont and Tirole (1988), and the third agent, represented by the market. In this second period the regulator may or may not make changes to the contract based on the information obtained in the first period, depending on whether she is committed or not. In the case of non-commitment, the firm has to take into account that the information it reveals - in addition to affecting its market value - may also lead the regulator to make changes in the contract. For simplicity, we will only address the second period in the non-commitment case, treating the commitment case as a one-period case.
In period one the regulator initially offers a set of contracts to the firm, which has private information about its own type \( \theta \in \{ \theta_L, \theta_H \} \). Both the market and the regulator have prior beliefs that \( \Pr[\theta = \theta_L] = v_1 \). By choosing a contract, the firm discloses some information for both the regulator and the market and, upon observing the realized cost, they update their beliefs about \( \theta \).

The posterior belief \( v_2 \) is the probability given by the agents for the firm to be efficient, given the contract it chooses. The posterior assumes values \( v^H \) or \( v^L \) when, respectively, the high or the low cost contracts are chosen in period one. Denoting by \( x_L \) and \( x_H \), respectively, the probabilities that the efficient and the inefficient firms choose the low cost contract, according to Bayes’ rule we have

\[
v^H = \frac{(1 - x_L)v_1}{(1 - x_L)v_1 + (1 - x_H)(1 - v_1)},
\]

and

\[
v^L = \frac{x_Lv_1}{x_Lv_1 + x_H(1 - v_1)}.
\]

Acting accordingly to this update, the short-term investors of the firm will trade their stocks based on its expected profit (defined by (9)), so the manager’s welfare is realized at

\[
U(\theta, \tilde{\theta}) = kU(\theta, \tilde{\theta}) + (1 - k)U^m(\tilde{\theta})
\]

where

\[
U^m(\theta_H) = v^H U(\theta_L, \theta_H) + (1 - v^H)U(\theta_H)
\]

\[
U^m(\theta_L) = v^L U(\theta_L) + (1 - v^L)U(\theta_H, \theta_L).
\]

In the second period, there are only long-term investors at the firm. The regulator, who also has a long term relationship with the firm, will revise the contract based on her posterior beliefs. The regulator problem will therefore be equivalent to the asymmetric information problem of the previous section.

It is also assumed that both the regulator and the firm’s manager discount the second-period welfare with factor \( \delta > 0 \). The regulator welfare is then

\[
W = S - (1 + \lambda)(t_1 + C_1) + U(\theta, \tilde{\theta}) + \delta[S - (1 + \lambda)(t_2 + C_2) + U_2(\theta, \tilde{\theta})]
\]

Finally, let us assume:

**A1.** The “true” (or “fundamental”) value of the firm \( U(\theta) \) cannot be negative.
Formally, an equilibrium in this setting can be defined as follows:

**Definition 1** Let the regulator’s strategy be an incentive scheme $C_1 \rightarrow t_1(C_1)$ in period 1, and $C_2 \rightarrow t_2(C_2; t_1(\cdot), C_1)$ in period 2. Assume that the reservation utility of the firm is zero, and thus the firm can choose not to participate in the relationship at any time in the game. The firm’s strategy is a choice of participation $\chi_\tau$ and effort $e_\tau$ in each period, and we let $\chi_\tau = 1$ if the firm decides to accept the incentive scheme in period $\tau$, and $\chi_\tau = 0$ otherwise. Thus, for each period the firm’s strategy is:

$$\{\chi_1(\theta, t_1(\cdot)), e_1(\theta, t_1(\cdot))\}$$

and

$$\{\chi_2(\theta, t_1(\cdot), t_2(\cdot), C_1), e_2(\theta, t_1(\cdot), t_2(\cdot), C_1)\}.$$

The strategies and beliefs above define a Perfect Bayesian Equilibrium (PBE) of the game between the regulator, the firm and the market if, and only if:

(I) $\{e_2(\cdot), \chi_2(\cdot)\}$ is optimal for the firm given $t_2(\cdot)$;

(II) $t_2(\cdot)$ is optimal to the regulator given his posterior beliefs;

(III) $\{e_1(\cdot), \chi_1(\cdot)\}$ is optimal for the firm given $t_1(\cdot)$ and that both the regulator’s incentive scheme for period 2 and the market value of the firm depends on $C_1$;

(IV) $t_1$ is optimal for the regulator given sequential strategies;

(V) the market value is defined given $C_1$ by the zero expected profit condition

$$U^m(\hat{\theta}) = E_\theta[U(\theta, \hat{\theta})|C_1 = C(\hat{\theta})],$$

(VI) both the market and the regulator posterior beliefs are equal and derived from the prior $v_1$, from (III), $t_1(\cdot)$ and $C_1$ using Bayes’ rule.

As mentioned earlier, the objective of our model is to understand how the interaction between the firm and the market could affect the regulator’s contract design. On the regulator-firm side, the model builds on the dynamic regulation model presented in Lafont and Tirole (1988). On the firm-market side, we introduce elements from the model presented in Miller and Rock (1985), as a way to endogenize the presence of the market in the firm’s decision. In the following subsections, we will initially present the case of
commitment, where the regulator can commit in the future to regulatory contracts offered in the first period. This will be our benchmark case. Next, we will analyze the case of non-commitment.

3.1 The Commitment Case

We define our commitment benchmark case as the case where $\delta = 0$, interpreted as having only one period. The problem faced by the regulator is very similar to the one presented in Section 2, and can be written as

$$\max_{t(\cdot), C(\cdot)} E_{\theta, \tilde{\theta}} [S - (1 + \lambda)(t(\tilde{\theta}) + C(\tilde{\theta})) + U(\theta, \tilde{\theta})]$$

subject to

$$U(\theta_L) \geq 0$$ (10)

$$U(\theta_H) \geq 0$$ (11)

$$U(\theta_L) \geq U(\theta_L, \theta_H)$$ (12)

$$U(\theta_H) \geq U(\theta_H, \theta_L).$$ (13)

In Section 4 we present our simulations for this and the more general non-commitment cases, with the additional assumption of quadratic costs. Our numerical results for the benchmark commitment case show that, for all sets of parameters used, the equilibrium is separating ($x^L = 1$), and, as in the model exposed in Section 2.2, effort of the efficient type is always the first best ($e_L = e^*$), as expected. Also, as we increase the presence of short-term investors, both the effort and transfers made to the inefficient type increase, while the transfer made to the efficient type decreases. One possible explanation for this effect is that an increase in the proportion of short-term investors in our model means that manager is more concerned about those investors, and so the direct profit gain from deviating would have a smaller effect on her welfare.

Additionally, not only the effort and transfer for the inefficient firm increases, but it also converges to the first best as $k$ approaches zero. Particularly, in the extreme case where $k$ equals zero, there would only be short-term market investors, and the manager would care only about the expected value of the firm. Thus, there would be no direct gains for the manager to deviate - the only source of gain would be the market value.
Anticipating this, the regulator will offer contracts where the firm’s profit is zero, for either type, and the first-best would be feasible.

This behaviour can be seen in Figure 1:

**Figure 1: Commitment case - numerical results (x-axis:k; y-axis: \( t_L, t_H, e_L, e_H \)) for \( \psi(e) = \frac{1}{2}e^2 \). Value of parameters:**

\[
S = 100, \theta_L = 1.5, \theta_H = 2, v = 0.5, \lambda = 0.1, \delta = 0.
\]

### 3.2 Non-commitment Case

In the non-commitment case, the parties cannot commit to a long-term contract. Instead, the relationship between the firm and the regulator is run by a series of short-term contracts, and the regulator might use information acquired from this relationship to expropriate the firm. In other words, whenever the firm reveals its type, the (non-committed) regulator will review its initial contract and leave the firm with no future rent. This is the basis of the dynamic regulation model presented in Laffont and Tirole (1988), which is a special case of our model, when there are only long-term investors holding the firm’s shares \((k = 1)\).
However, as explained in the beginning of this section, our model has a third participant to this relationship: the market. Since the firm’s manager is concerned with both short-term investors (who hold a fraction \((1 - k)\) of the firm’s shares) and long-term investors, there is a conflict between the information the firm wants to reveal to the market and to the regulator. Ideally, the efficient firm would want to show to the market that it is operationally efficient (since the market trades based on the firm’s expected value), while not fully revealing its type to the regulator (since it could be expropriated later on). The main purpose is to analyze how the presence of the market affects the relationship between the regulator and the regulated firm in both cases.

For the second period, \(W^{FI}(v)\) and \(W^{AI}(v)\) are the optimal expected one-period welfare under full information and under incomplete information, respectively, for prior \(v\). For the two-type case we are investigating, we assume there are two contracts being offered by the regulator, \((t_L, C_L)\) and \((t_H, C_H)\). Thus, the regulator maximizes the following expected welfare:

\[
W^{NC} = S - v_1 x_L [(1 + \lambda) (t_L + C_L) - \mathcal{U}(\theta_L, \theta_L)] - v_1 (1 - x_L) [(1 + \lambda) (t_H + C_H) + \mathcal{U}(\theta_L, \theta_H)] - (1 - v_1) x_H [(1 + \lambda) (t_L + C_L) + \mathcal{U}(\theta_H, \theta_L)] - (1 - v_1) (1 - x_H) [(1 + \lambda) (t_H + C_H) + \mathcal{U}(\theta_H, \theta_H)] + \delta \left\{ [v_1 x_L + (1 - v_1) x_H] W^{AI}(v^L) + [v(1 - x_L) + (1 - v_1) (1 - x_H)] W^{AI}(v^H) \right\},
\]

subject to

\[
\begin{align*}
\mathcal{U}(\theta_L) &\geq 0 \quad (14) \\
\mathcal{U}(\theta_H) &\geq 0 \quad (15) \\
\mathcal{U}(\theta_L) + \delta \Phi(e_H(v_L)) &\geq \mathcal{U}(\theta_L, \theta_H) + \delta \Phi(e_H(v_H)) \quad (16) \\
\mathcal{U}(\theta_H) &\geq \mathcal{U}(\theta_H, \theta_L), \quad (17)
\end{align*}
\]

where \(\Phi(e_H(v_2))\) is the informational rent left for the efficient firm at the second period to induce revelation, defined by equation [8].

One perverse effect generated by the lack of commitment from the regulator is clear from the intertemporal incentive compatibility for type \(\theta_L\) (equation [16]). Since, by revealing itself, the efficient firm will be expropriated at the second period \((e_H(v_L)\) is decreasing toward zero), there are more incentives for the efficient firm to choose the inefficient contract. It is also important to note that the rent for the inefficient type
at the second period will always be zero, even if the contracts are separating and it pretends to be efficient. This happens because, without commitment, the firm can leave the relationship after the first period, in a take-the-money-and-run strategy.

The possibility of this kind of action by the inefficient firm is the cause of greater part of the complexity of the problem, and induces some types of equilibrium where both the ICs are binding, when regularly only the incentive compatibility for type $\theta_L$ is binding. This happens because, to generate incentives for the efficient firm to exert effort, compensating the second period expropriation, the principal has to increase the transfers made to the efficient firm, making the efficient contract more attractive for the inefficient firm, who can take the transfers and leave the relationship before having to exert first-best efforts at the second period.

On the other hand, in the same direction as in the commitment case, it is expected for the presence of the market to act to counterbalance these negative effects of the lack of commitment. In fact, the higher the market presence, the smaller are the gains the efficient firm will have deviating from its contract, since more of the stockholders will believe it to be an inefficient firm.

4 Example and Comparative statics

To analyze the behavior of the model described above, we do some comparative statics in the special case where the disutility of effort has a quadratic form: $\psi(e) = \frac{1}{2} \left[ \max(0, e) \right]^2$.\footnote{There is a similar numerical exercise in Laffont and Tirole (1993), so our results could, to some extent, be compared to theirs. Note that, when $k = 1$ (i.e. in the absence of the market), both models are equivalent.} We will use this special case to study numerically the effect of the presence of the market in our model. All numerical simulations where run with the software Matlab R2012b, as detailed on Appendix C.

To proceed with our analysis, we chose a set of parameters to define a basis non-commitment case and analyzed the equilibrium behaviour for different levels of market participation (i.e., as $k$ varies from 0 to 1). Table I shows the numerical results.
In Table 1, it is possible to see that the presence of the market leads to more powerful incentives, inducing more effort from the inefficient firm. Moreover, the rent of the efficient firm decreases and welfare increases as the share of short-term market investors increases (k decreases). For this set of parameters, the best equilibrium is always a separating one, regardless of the amount of market investors in the firm.

From this benchmark equilibrium, we vary some of the parameters to understand their effects on the equilibrium. As a first exercise, we evaluate our model when we vary the intertemporal discount rate δ. As a general result, for small, positive δ the best equilibrium is still a separating equilibrium, but as δ increases, we find that the equilibrium starts to show some degree of pooling or a semiseparation (x_L < 1). However, when the participation of the market increases, the separating equilibrium continues to dominate, even for larger discount factors (δ = 10, shown in Table 2). Table 2 shows some results for different values of discount factors and proportions of long-term investors (k).

9Notice that δ = 0 corresponds to the static case.
Based on these results, the presence of the market induces more separation - for a fixed value of $\delta$, separating equilibrium dominates as $k$ decreases. Also, welfare increases and the informational rent of the efficient type decreases when the proportion of shares in the market increases. The effort of the inefficient type also increases in this case, showing again that the presence of the market leads to more powerful contracts.

Also, we analyze the effect of varying the difference between the technological parameters for the types, $\Delta \theta$, in the equilibrium. As a general rule, the equilibrium tends more towards a pooling as $\Delta \theta$ decreases. However, we see some interesting results as we vary the amount of market participation: for a given $\Delta \theta$, the increased participation of short-term market investors leads to more separation. Putting this differently, separating equilibrium was dominant for $k < 0.5$ in all cases analyzed. This means that the presence of the market might induce separation, even when $\Delta \theta$ is too small (we analyzed the case for $\Delta \theta = 0.01$) - which can viewed as a positive effect of the market. However, as $\Delta \theta$ increases, we see that the presence of the market becomes less important, since separation is always optimal for $\Delta \theta$ large enough. Tables 3 and 4 show those results.
We can summarize our main results in the following statements:

I. In a separating equilibrium, the presence of the market increases welfare, reduces informational rent and leads to more powerful incentives.

II. Whenever there is a separating equilibrium for \( k^\ast \), the equilibrium is separating for all \( k \leq k^\ast \).

The results of our numerical exercise show that the presence of the market can be beneficial for regulation. It appears that, due to the signaling effect, the market induces more revelation and the regulator can potentially reduce the amount of informational rent left to the firm. In this sense, differently from Faure-Grimaud (2002), we view the
presence of the market as complementary to regulation, as it helps the regulator to achieve a more efficient outcome.

Lastly, by changing the values of the prior probability $v$ and of the cost of public funds $\lambda$ we have separating equilibria for all cases, for all values of $k$. Thus, the results for changes in those parameters are as expected from the theory: as $v_1$ increases, the amount of informational rent left to the efficient firm to reveal itself decreases, since the regulator has a stronger prior belief that the firm’s type is efficient; as we vary the cost of public funds $\lambda$, both expected rent and welfare decrease when $\lambda$ decreases, and the effort level for the inefficient type is also reduced. Clearly, as public funds become more expensive, it is more costly for the regulator to make transfers, and thus the level of efforts is lower.

Regarding the effect of the market for varying the prior belief $v$ or the cost of public funds ($\lambda$), informational rent decreases and welfare increases as $k$ approaches zero - which corroborates our previous results regarding the effect of the presence of market, namely that the presence of the market seems to have a positive effect on welfare. Appendix C shows some simulation results for different values of $v_1$ and $\lambda$.

5 Conclusion

Our main objective in this paper was to analyze how the presence of the market, when the market has access to the same information about the firm as the regulator, could change the relationship between the regulator and the regulated firm. For this purpose, we used a signaling model, where the level of information disclosure of the firm was endogenous, and the firm could choose whether or not to reveal itself. Our basic premise was that the firm wants to appear efficient to the market, and inefficient to the regulator.

As shown in our results, the presence of the market does affect the relationship between the firm and the regulator. The regulator benefits from the presence of the market mainly through more powerful regulation contracts, which leads to increased welfare. This can indicate that the public listing of regulated companies can indeed induce more separation and that the manager’s concern about the market can help the regulator achieve better outcomes.
References


Appendix

A Commitment

Consider

\[ U(\theta_j) = kU_j + (1 - k)E[U(\theta) \mid C = C_j] \]

where

\[ U_j = t_j - \psi(\theta_j - C_j), \]

for \( j = L, H \). We then have

\[ U_L = kU_L + (1 - k) \left[ v^L U_L + (1 - v^L)U(\theta_H, \theta_L) \right] \]
\[ U_H = kU_H + (1 - k) \left[ v^H U(\theta_L, \theta_H) + (1 - v^H)U_H \right]. \]

We also know that the deviation payoff of both types (i.e. \( U(\theta, \hat{\theta}) \), where \( \hat{\theta} \) is the announcement made by type \( \theta \)) in terms of the truthful announcement and the rent can be written as: when type \( \theta_L \) announces \( \theta_H \), we have

\[
U(\theta_L, \theta_H) = k[t_H - \psi(e_H - \Delta \theta)] + (1 - k)E[U(\cdot) \mid C = C_H] \\
= k[t_H - \psi(e_H - \Delta \theta)] - k[t_H - \psi(e_H)] \\
+ k[t_H - \psi(e_H)] + (1 - k)E[U(\cdot) \mid C = C_H] \\
= U(\theta_H) + k\Phi(e_H).
\]

Similarly, when type \( \theta_H \) announces \( \theta_L \), we have

\[
U(\theta_H, \theta_L) = k[t_L - \psi(\theta_H - c_L)] + (1 - k)E[U(\cdot) \mid C = C_L] \\
= k[t_L - \psi(\theta_H - c_L)] - k[t_L - \psi(\theta_L - C_L)] \\
+ k[t_L - \psi(\theta_L - c_L)] + (1 - k)E[U(\cdot) \mid C = C_L] \\
= U(\theta_L) - k\Phi(e_L + \Delta \theta). 
\]
Thus, we can write \( IC_L \) and \( IC_H \), respectively, as

\[
U_L \geq U_H + k\Phi(e_H) \\
U_H \geq U_L - k\Phi(e_L + \Delta \theta).
\]

Also, it is trivial to show that constraints \( IC_L \) and \( IR_H \) together (equations (12) and (11)) imply constraint \( IC_H \) (equation (13)), and that Assumption 1 imply \( U_H \geq 0 \). Adding constraints \( IC_L \) and \( IC_H \) together, we have

\[
k\Phi(e_H) \leq IC_L U_L - U_H \leq IC_H k\Phi(e_L + \Delta \theta),
\]

which implies that \( C_L \leq C_H \). We thus have

\[
0 \geq k [\Phi(e_H) - \Phi(e_L + \Delta \theta)],
\]

which is true if and only if \( C_L \leq C_H \). Particularly, \( IC_H \) is slack whenever \( C_L < C_H \) - and, in this case, \( x_H = 0 \). Notice also that \( IC_L \) is binding, otherwise the regulator would be leaving too much rent to the efficient firm.

B Non-commitment

In this Appendix, we make some calculations similar to those done in Laffont and Tirole (1988). We calculate the expressions for equilibria type I, II and III for our model. Differently from theirs, our model has a third agent, the short-term investors in the market, as described in Section 3.

**Equilibrium Type I:** For this equilibrium, the inefficient type plays \( C_H \) with probability 1 (remember that only the \( IC_L \) constraint is binding). Thus, we have

\[
x_H = \text{Pr}[C = C_L \mid \theta = \theta_H] = 0 \Rightarrow v^L = \text{Pr}[\theta = \theta_L \mid C = C_L] = 1.
\]

This means that, whenever the market observes \( C_L \) being played in this equilibrium, it knows with certainty that this is the efficient type. We thus have that

\[
E_{\theta}[U(\theta) \mid C_L] = U(\theta_L) = U(\theta_H).
\]

Also, when the market observes a signal \( C_H \), we have that

\[
E_{\theta}[U(\theta) \mid C = C_H] = v^H U(\theta_L, \theta_H) + (1 - v^H) U(\theta_H).
\]
From the binding incentive compatibility constraint (equation (16)), we have that
\[ U(\theta_L) = [k + (1 - k)v^H] \Phi(e_H) + \delta [\Phi(e(v_H)) - \Phi(e(v_L))] \] (18)
and
\[ t_L = \psi(e_L) + U(\theta_L). \] (19)
From Assumption 1 we have that \( U(\theta_H) = 0 \), thus
\[ t_H = \psi(e_H) \] (20)
and \( U(\theta_H) = (1 - k)v^H U(\theta_L, \theta_H) \), or
\[ U(\theta_H) = (1 - k)v^H \Phi(e_H). \] (21)

We are now ready to calculate the expected welfare for a type I equilibrium:
\[ W^I = S - (1 + \lambda) [v_1x_L (t_L + C_L) + v_1(1 - x_L) (t_H + C_H) + (1 - v_1) (t_H + C_H)] \\
+ v_1x_L U(\theta_L) + v_1(1 - x_L) U(\theta_L, \theta_H) + (1 - v_1) U(\theta_H) \\
+ \delta \{ v_1x_L W^{FI}(1) + [v_1(1 - x_L) + (1 - v_1)] W^{AI}(v^H) \} \]
Substituting equations (18)-(21) and rearranging we have
\[ W^I = S - (1 + \lambda) [v_1x_L (\psi(e_L) + C_L) - (1 + \lambda)v_1(1 - x_L) [\psi(e_H - \Delta \theta) + C_H] \\
- (1 + \lambda)(1 - v_1) [\psi(e_H) + C_H] + \{v_1(1 - x_L) [k - (1 + \lambda)] - \lambda v_1x_L k\} \Phi(e_H) \\
+ (1 - k)v^H [v_1(1 - x_L) + (1 - v_1) - \lambda v_1x_L] \Phi(e_H) \\
+ \delta \{ v_1x_L W^{FI}(1) + [v_1(1 - x_L) + (1 - v_1)] W^{AI}(v^H) \} \\
- \lambda v_1x_L \delta [\Phi(e(v^H)) - \Phi(e(v^L))] \].
This is the welfare function for a type I equilibrium.

**Equilibrium Type II:** In a type II equilibrium, it is the efficient type that chooses his own contract with probability 1. In this case we have
\[ x_L = \Pr[C = C_L \mid \theta = \theta_L] = 1 \Rightarrow v^H = \Pr[\theta = \theta_L \mid C = C_H] = 0. \]

Here we have only the incentive compatibility constraint for the inefficient type (equation (17)) binding and, when the signal observed is \( C_H \), the market knows with certainty that it is the inefficient type playing:
\[ E_\theta[U(\theta)|C = C_H] = U(\theta_H) = \mathcal{U}(\theta_H). \]

When the signal \( C_L \) is observed, we have

\[ E_\theta[U(\theta) | C = C_L] = v^L U(\theta_L) + (1 - v^L) U(\theta_H, \theta_L). \]

From the individual rationality constraint for the inefficient type binding we know that

\[ \mathcal{U}(\theta_H) = U(\theta_H) = 0 \]  
(22)

and

\[ t_H = \psi(e_H). \]  
(23)

Also, from the incentive compatibility of the inefficient type also binding, we have

\[ \mathcal{U}(\theta_H, \theta_L) = 0 \]

which implies that

\[ \mathcal{U}(\theta_L) = k \Phi(e_L + \Delta \theta). \]  
(24)

Finally, we know that

\[ \mathcal{U}(\theta_L) = [k + (1 - k)v^L] U(\theta_L) + (1 - k)(1 - v^L) U(\theta_H, \theta_L), \]

and we find

\[ U(\theta_L) = \frac{k + (1 - k)(1 - v^L)}{k + (1 - k)(1 + v^H - v^L)} \Phi(e_L + \Delta \theta) \]

and

\[ t_L = \psi(e_L) + \frac{k + (1 - k)(1 - v^L)}{k + (1 - k)(1 + v^H - v^L)} \Phi(e_L + \Delta \theta). \]  
(25)

We now can express the welfare function for a type II equilibrium

\[ W^{II} = S - (1 + \lambda) [v_1 (t_L + C_L) + (1 - v_1)x_H (t_L + C_L)] \]

\[-(1 + \lambda)(1 - v_1)(1 - x_H) (t_H + C_H) \]

\[ + v_1 \mathcal{U}(\theta_L) + (1 - v_1)x_H \mathcal{U}(\theta_H, \theta_L) + (1 - v_1)(1 - x_H) \mathcal{U}(\theta_H) \]

\[ + \delta \left\{ [v_1 + (1 - v_1)x_H] W^{AI}(v^L) + [(1 - v_1)(1 - x_H)] W^{FI}(0) \right\}. \]
and, after substituting equations (22)-(25) we find

\[
W^{II} = S - (1 + \lambda)v_1 [\psi(e_L) + C_L] - (1 + \lambda)(1 - v_1)x_H [\psi(e_L + \Delta\theta) + C_L] \\
-(1 + \lambda)(1 - v_1)(1 - x_H) [\psi(e_H) + C_H] \\
+ [(1 + \lambda)(1 - v_1)x_H + v_1 k] \Phi(e_L + \Delta\theta) \\
-(1 + \lambda)(1 - v_1)(1 - x_H) [\psi(e_H) + C_H] \\
+ \delta \left\{ [v_1 + (1 - v_1)x_H] W^{AI}(v^L) + [(1 - v_1)(1 - x_H)] W^{FI}(0) \right\}.
\]

This is the expression for the welfare in a type II equilibrium.

**Equilibrium type III:** In a type III equilibrium, the incentive compatibility for both types are binding, so the market cannot identify the player for either signal. Thus

\[
E_\theta [U(\theta) | C = C_L] = v^L U(\theta_L) + (1 - v^L) U(\theta_H, \theta_L) \\
E_\theta [U(\theta) | C = C_H] = v^H U(\theta_L, \theta_H) + (1 - v^H) U(\theta_H),
\]

where the posteriors are

\[
v^H = \frac{v_1(1 - x_L)}{(1 - v_1)(1 - x_H) + v_1(1 - x_L)} \\
v^L = \frac{v_1 x_L}{x_H(1 - v_1) + v_1 x_L}.
\]

From Assumption 1 and optimality, we know that \(U(\theta_H) = 0\), which implies that

\[
U(\theta_L) = [k + (1 - k)v^L] U(\theta_L) + (1 - k)(1 - v^L) U(\theta_H, \theta_L)
\]

and

\[
U(\theta_H) = (1 - k)v^H \Phi(e_H).
\]  (26)

From the incentive compatibility of the efficient type binding, and from equation the expression for \(U(\theta_L, \theta_H)\) in Appendix A we have

\[
U(\theta_L) = U(\theta_H) + k \Phi(e_H) + \delta \left[ \Phi(e(v^H)) - \Phi(e(v^L)) \right]
\]  (27)

which implies, by substituting equations (26) and (27) that
\[ [k + (1 - k)v^L]U(\theta_L) + (1 - k)(1 - v^L)U(\theta_H, \theta_L) \]
\[ = [k + (1 - k)v^H] \Phi(e_H) + \delta \left[ \Phi(e (v^H)) - \Phi(e (v^L)) \right]. \quad (28) \]

Also, from incentive compatibility of the inefficient type binding, and from the expression for \( U(\theta_H, \theta_L) \), we have

\[ U(\theta_H) = U(\theta_H, \theta_L) = U(\theta_L) - k \Phi(e_L + \Delta \theta). \]

Then, using equations (26) and (27), and knowing that \( U(\theta_L, \theta_H) = \Phi(e_H) \), we find

\[ [k + (1 - k)v^L]U(\theta_L) + (1 - k)(1 - v^L)U(\theta_H, \theta_L) = (1 - k)v^H \Phi(e_H) + k \Phi(e_L + \Delta \theta). \quad (29) \]

Therefore, for both incentive constraints to bind it must be that equations (28) and (29) hold. This happens if, and only if

\[ \delta \left[ \Phi(e (v^H)) - \Phi(e (v^L)) \right] = k \left[ \Phi(e_L + \Delta \theta) - \Phi(e_H) \right]. \]

To solve for \( t_L \), we use the first equation in the system above and find

\[ t_L = [k + (1 - k)v^L]\psi(e_L) + (1 - k)(1 - v^L)\psi(e_L + \Delta \theta) \]
\[ + \left[ (1 - k)v^H + k \right] \Phi(e_H) + \delta \left[ \Phi(e (v^H)) - \Phi(e (v^L)) \right]. \quad (30) \]

Finally, we substitute \( t_L \) into \( U(\theta_L) = t_L - \psi(e_L) \) and \( U(\theta_H, \theta_L) = t_L - \psi(e_L + \Delta \theta) \), rearrange, and find

\[ U(\theta_L) = [(1 - k)(1 - v^L)] \Phi(e_L + \Delta \theta) \]
\[ + \left[ (1 - k)v^H + k \right] \Phi(e_H) + \delta \left[ \Phi(e (v^H)) - \Phi(e (v^L)) \right] \quad (31) \]

and

\[ U(\theta_H, \theta_L) = [(1 - k)v^H + k] \Phi(e_H) - [(1 - k)v^L + k] \Phi(e_L + \Delta \theta) \]
\[ + \delta \left[ \Phi(e (v^H)) - \Phi(e (v^L)) \right]. \quad (32) \]

We can now substitute equations (26), (27), (30), (31) and (32) in the regulator’s
objective function for a type III equilibrium. The regulator maximizes

\[ W^{III} = S - (1 + \lambda)v_1 x_L [\psi(e_L) + c_L] - (1 + \lambda)v_1 (1 - x_L) [\psi(e_H - \Delta \theta) + c_H] \\
- (1 + \lambda)(1 - v_1) x_H [\psi(e_L + \Delta \theta) + c_L] - (1 + \lambda)(1 - v_1)(1 - x_H) [\psi(e_H) + c_H] \\
- \lambda v_1 x_L k \Phi(\psi(e_L + \Delta \theta) + v_1(1 - x_L) [k - (1 + \lambda)] \Phi(e_H) \\
+ (1 - k)(1 + \lambda)(1 - v_1) x_H \Phi(e_L + \Delta \theta) \\
+ (1 - k) v^H \{ (1 - (1 + \lambda) [(1 - v_1) x_H + v_1 x_L] \Phi(e_L) \\
- (1 - k)(1 - v^L)(1 + \lambda) [(1 - v_1) x_H + v_1 x_L] \Phi(e_L + \Delta \theta) \\
+ \delta \{ [v_1 x_L + (1 - v_1) x_H] W^{AI}(v^L) + [v_1(1 - x_L) + (1 - v_1)(1 - x_H)] W^{AI}(v^H) \} \}

s.t. \[ k \Phi(\psi(e_L + \Delta \theta) - \Phi(e_H)] = \delta \left[ \Phi(e(v^H)) - \Phi(e(v^L)) \right] \]

\section{C Numerical Simulations}

As mentioned in Section 4, all numerical simulations were implemented with the use of the software Matlab R2011b. In this appendix we outline the procedure used to find the equilibrium of the dynamic problem and the results for different sets of parameters.

For each case, including the commitment ($\delta = 0$), the problem was solved for all three types of equilibrium, as defined in the Appendix B. For each type of equilibrium and randomization strategies, it is possible to find the optimal contract for the regulator - so the welfare function can be rewritten in terms of the exogenous variables ($S, \theta_L, \theta_H, \lambda, v$ and $\delta$) and randomization profiles. Those welfare functions were then numerically optimized for $x_L$ and $x_H$ using Matlab’s solver \textit{fmincon} interior-point algorithm with several uniformly random starting points.
### Table 5

**Comparative statics for lambda - 1**

<table>
<thead>
<tr>
<th>$\lambda$ = 0.01</th>
<th>$k$ = 0.01</th>
<th>$k$ = 0.2</th>
<th>$k$ = 0.4</th>
<th>$k$ = 0.6</th>
<th>$k$ = 0.8</th>
<th>$k$ = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^{NC}$</td>
<td>144.48</td>
<td>144.48</td>
<td>144.48</td>
<td>144.48</td>
<td>144.48</td>
<td>144.48</td>
</tr>
<tr>
<td>$x_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x_H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$e_H$</td>
<td>0.9999</td>
<td>0.9980</td>
<td>0.9960</td>
<td>0.9940</td>
<td>0.9920</td>
<td>0.9900</td>
</tr>
<tr>
<td>$t_L$</td>
<td>0.7549</td>
<td>0.8496</td>
<td>0.9484</td>
<td>1.0464</td>
<td>1.1436</td>
<td>1.2400</td>
</tr>
<tr>
<td>$t_H$</td>
<td>0.4999</td>
<td>0.4980</td>
<td>0.4940</td>
<td>0.4940</td>
<td>0.4921</td>
<td>0.4901</td>
</tr>
<tr>
<td>Rent</td>
<td>0.2549</td>
<td>0.3496</td>
<td>0.4484</td>
<td>0.5464</td>
<td>0.6436</td>
<td>0.7400</td>
</tr>
</tbody>
</table>

Value of parameters: $S = 100, \theta_L = 1, \theta_H = 2, v = 0.5, \lambda = 0.01, \delta = 0.5$

### Table 6

**Comparative statics for lambda - 2**

<table>
<thead>
<tr>
<th>$\lambda$ = 1</th>
<th>$k$ = 0.01</th>
<th>$k$ = 0.2</th>
<th>$k$ = 0.4</th>
<th>$k$ = 0.6</th>
<th>$k$ = 0.8</th>
<th>$k$ = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^{NC}$</td>
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<td>146.77</td>
<td>146.75</td>
<td>146.75</td>
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<tr>
<td>$x_L$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x_H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$e_H$</td>
<td>0.995</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
</tr>
<tr>
<td>$t_L$</td>
<td>0.7549</td>
<td>0.83</td>
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<td>0.87</td>
<td>0.83</td>
<td>0.75</td>
</tr>
<tr>
<td>$t_H$</td>
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<td>0.32</td>
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</tr>
<tr>
<td>Rent</td>
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<td>0.37</td>
<td>0.37</td>
<td>0.33</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Value of parameters: $S = 100, \theta_L = 1, \theta_H = 2, v = 0.5, \lambda = 1, \delta = 0.5$

### Table 7

**Comparative statics for $v_1$ - 1**

<table>
<thead>
<tr>
<th>$v$ = 0.1</th>
<th>$k$ = 0.01</th>
<th>$k$ = 0.2</th>
<th>$k$ = 0.4</th>
<th>$k$ = 0.6</th>
<th>$k$ = 0.8</th>
<th>$k$ = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^{NC}$</td>
<td>147.68</td>
<td>147.68</td>
<td>147.68</td>
<td>147.68</td>
<td>147.68</td>
<td>147.68</td>
</tr>
<tr>
<td>$x_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x_H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$e_H$</td>
<td>0.9998</td>
<td>0.9979</td>
<td>0.9959</td>
<td>0.9939</td>
<td>0.9919</td>
<td>0.9898</td>
</tr>
<tr>
<td>$t_L$</td>
<td>0.7549</td>
<td>0.8495</td>
<td>0.9484</td>
<td>1.0463</td>
<td>1.1435</td>
<td>1.2398</td>
</tr>
<tr>
<td>$t_H$</td>
<td>0.4999</td>
<td>0.4979</td>
<td>0.4959</td>
<td>0.4939</td>
<td>0.4919</td>
<td>0.4899</td>
</tr>
<tr>
<td>Rent</td>
<td>0.2549</td>
<td>0.3495</td>
<td>0.4483</td>
<td>0.5463</td>
<td>0.6435</td>
<td>0.7398</td>
</tr>
</tbody>
</table>

Value of parameters: $S = 100, \theta_L = 1, \theta_H = 2, v = 0.1, \lambda = 0.1, \delta = 0.5$
Table 8

Comparative statics for $v_1 - 2$

<table>
<thead>
<tr>
<th>$v = 0.7$</th>
<th>$k = 0.01$</th>
<th>$k = 0.2$</th>
<th>$k = 0.4$</th>
<th>$k = 0.6$</th>
<th>$k = 0.8$</th>
<th>$k = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^{NC}$</td>
<td>146.87</td>
<td>146.83</td>
<td>146.79</td>
<td>146.77</td>
<td>146.75</td>
<td>146.75</td>
</tr>
<tr>
<td>$x_L$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$x_H$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$e_L$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$e_H$</td>
<td>0.9978</td>
<td>0.9575</td>
<td>0.9151</td>
<td>0.8727</td>
<td>0.8303</td>
<td>0.7878</td>
</tr>
<tr>
<td>$t_L$</td>
<td>0.7549</td>
<td>0.8415</td>
<td>0.9160</td>
<td>0.9736</td>
<td>1.0142</td>
<td>1.0378</td>
</tr>
<tr>
<td>$t_H$</td>
<td>0.4978</td>
<td>0.4584</td>
<td>0.4187</td>
<td>0.3808</td>
<td>0.3447</td>
<td>0.3103</td>
</tr>
<tr>
<td>$Rent$</td>
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<td>0.3415</td>
<td>0.4160</td>
<td>0.4736</td>
<td>0.5142</td>
<td>0.5378</td>
</tr>
</tbody>
</table>

Value of parameters: $S = 100, \theta_L = 1, \theta_H = 2, v = 0.7, \lambda = 0.1, \delta = 0.5$